A Parallel Architecture for the Generalized Traveling Salesman Problem

Max Scharrenbroich
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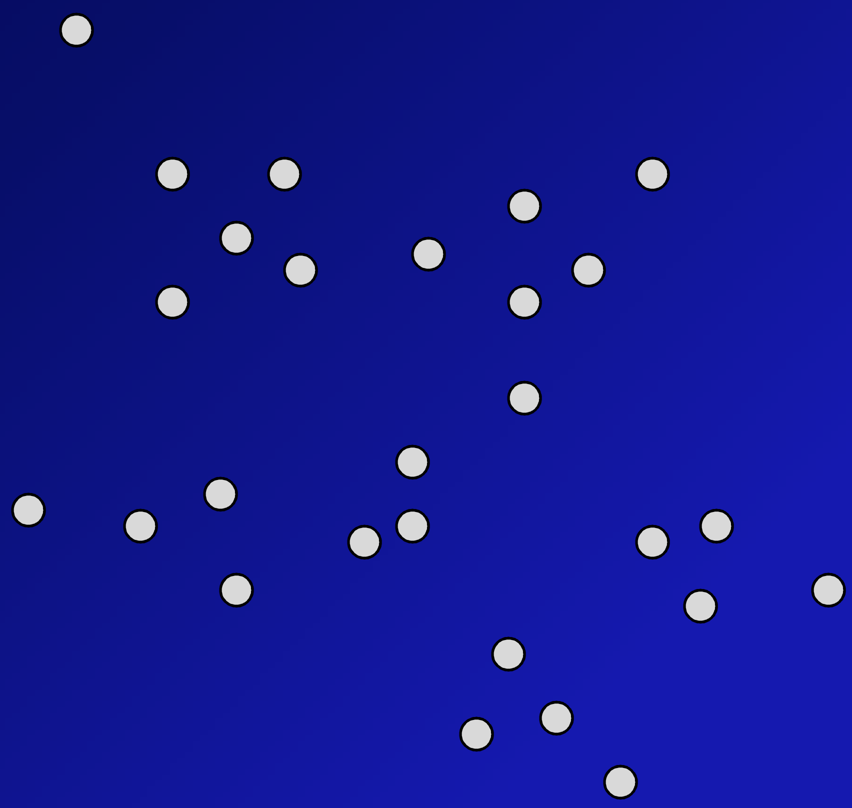
Advisor: Dr. Bruce L. Golden
R. H. Smith School of Business

Background and Introduction

- What is the Generalized Traveling Salesman Problem (GTSP)?
  - Variation of the well-known traveling salesman problem.
  - A set of nodes to be visited is partitioned into clusters.
  - Objective: Find a minimum-cost tour visiting exactly one node in each cluster.
  - Example on the following slides…

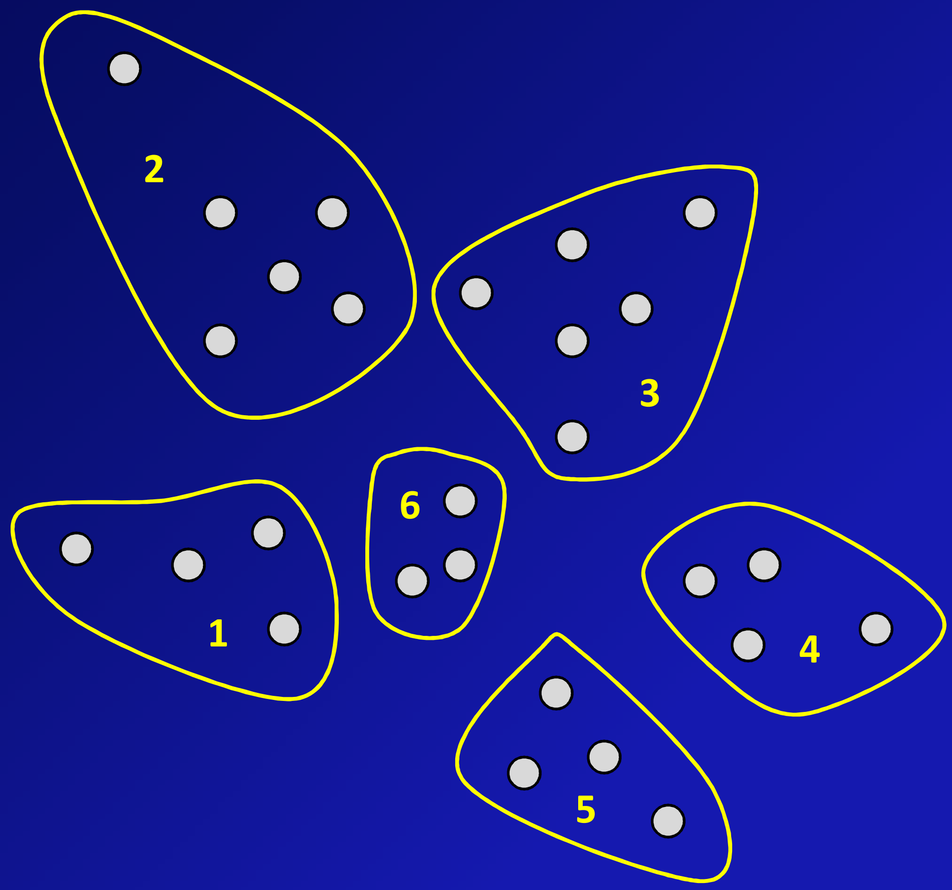
GTSP Example

- Start with a set of nodes or locations to visit.



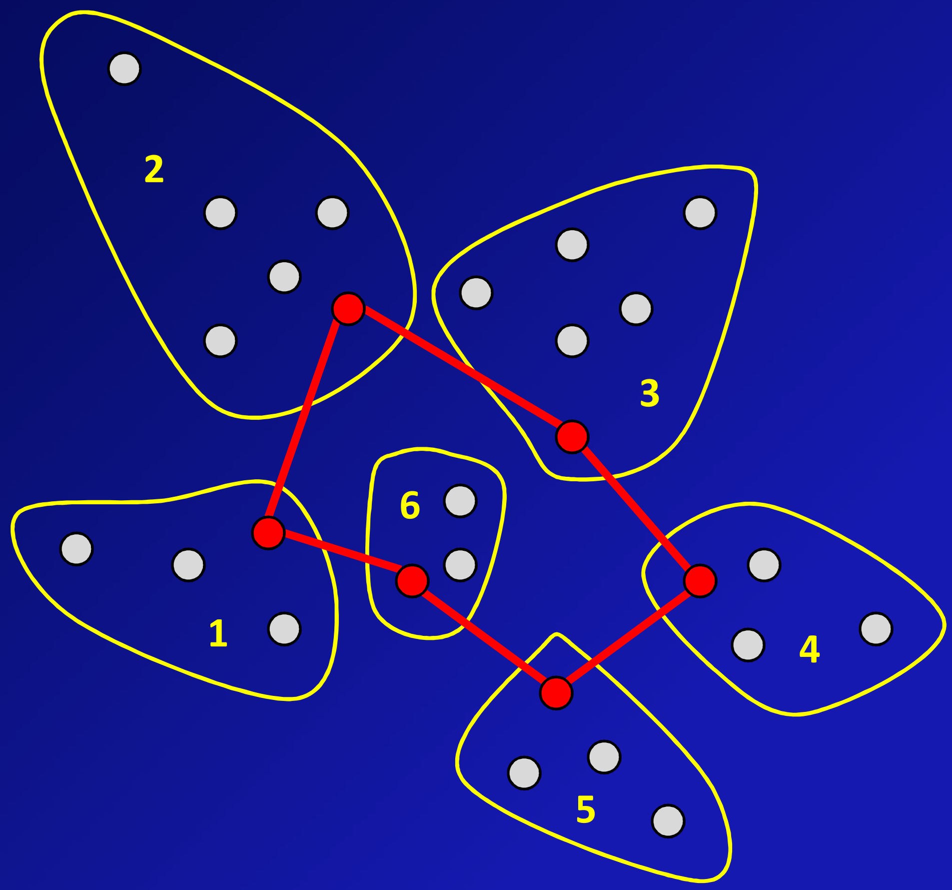
GTSP Example (continued)

- Partition the nodes into clusters.



GTSP Example (continued)

- Find the minimum tour visiting each cluster.



Applications

- The GTSP has many real-world applications in the field of routing:
  - Mailbox collection and stochastic vehicle routing.
  - Warehouse order picking with multiple stock locations.
  - Airport selection and routing for courier planes.

Mathematical Formulation

- The GTSP can be formulated as an Integer Linear Program (ILP).
- Graph: $G(V, E)$, where V is a set of vertices partitioned into m clusters $\{V_1, V_2, \ldots V_m\}$ and E is a set of edges connecting the vertices.
- Distance Matrix: C is a distance matrix defined on E, where c_e is the weight of edge e.
- Decision Variables: x_e and y_v are 0-1 decision variables representing the solution edges and vertices respectively.

$$\min \sum_{e \in E} c_e x_e$$

Subject to:

$$\sum_{v \in V_k} y_v = 1 \qquad k = 1,2,\ldots m$$

$$\sum_{e \in \delta(v)} x_e = 2y_v \qquad \text{for} \quad v \in V$$

$$\sum_{e \in \delta(S)} x_e \geq 2y_v \qquad S \subset V_*, v \in V_* - S$$

Algorithms for the GTSP

- Like the TSP, the GTSP is NP-hard.
- There exist exact algorithms that rely on smart enumeration techniques:
  - Brand-and-cut (B&C) algorithm (M. Fischetti, 1997)
  - Provided exact solutions to reasonably sized GTSP problems ($48 \le n \le 442$ and $10 \le m \le 89$).
  - For problems with larger than 90 clusters the run time of the B&C algorithm began approaching one day.

Algorithms for the GTSP (continued)

- Heuristic approaches to the GTSP:
  - Generalized Nearest Neighbor Heuristic (C.E. Noon, 1988)
  - Random-key Genetic Algorithm (L. Snyder and M. Daskin, 2006)
  - mrOX Genetic Algorithm (J. Silberholz and B.L. Golden, 2007)*

Overview of Genetic Algorithms (GA)

- Proposed in the 1970’s by Holland.
- Stochastic search technique commonly used to find approximate solutions to combinatorial optimization problems.
- Inspired by the process of natural selection and the theory of evolutionary biology.
- Simulate the evolution of a population of solutions.

Components of a GA >

Overview of GAs (continued)

- Components of Genetic Algorithms:
- Selection Operator:
  - Select the best solutions (chromosomes) for breeding.
- Crossover Operator:
  - Combine the pairs of selected solutions in some way to produce new solutions.
- Mutation Operator:
  - Randomly modify some solutions to preserve population diversity.
- Termination Criteria:
  - Terminate after a number of iterations (or period of time).
  - Terminate after a better solution is not found within a number of generations

Overview of mrOX Genetic Algorithm

- First, what is mrOX?
- The mrOX is the crossover operator at the heart of the mrOX GA.
- Proposed by J. Silberholz and B.L. Golden (2007).
- Modified rotational ordered crossover operator.
- Modification of the TSP ordered crossover (OX) proposed by (Davis, 1985).
- Results in a more “intelligent” crossover than the OX.

Example of OX >

Example of the GTSP OX

- Chromosomes are represented by path-lists.

Start building child chromosome with P1 sub-path

Final OX chromosome

P1 [$1^2, 3^5, 2^3, 4^8$]

P1' [1^2 | $3^5, 2^3$ | 4^8] → [- | $3^5, 2^3$ | -] [- | $3^5, 2^3$ | 1^1] [4^7 | $3^5, 2^3$ | 1^1]

{ - } { 1 } { 1, 4 }

Add genetic material from P2

P2' [2^4 | $3^5, 4^7$ | 1^1]

Inserted sub-path from P2

Randomly generate cut-points

P2 [$2^4, 3^5, 4^7, 1^1$]

1^2 = (cluster)$^{(\text{node})}$

mrOX

- Modify the inserted sub-path resulting from the OX operator and find the best one.
- rOX – rotational + OX:
  - Creates rotations and reversals of the inserted sub-path.
  - Example sub-tour: {1, 2, 3}
    - Rotations: { {1, 2, 3} {2, 3, 1} {3, 1, 2} }
    - Reversals: { {3, 2, 1} {1, 3, 2} {2, 1, 3} }
- mrOX – modified + rOX:
  - For each set of sub-paths generated in rOX create combinations of each node in the clusters at the end-points.
  - $\{ 1^{\{A, B\}}, 3, 2^{\{D, E\}} \}$:
    - $\{1^{A}, 3, 2^{D}\}$ $\{1^{A}, 3, 2^{E}\}$ $\{1^{B}, 3, 2^{D}\}$ $\{1^{B}, 3, 2^{E}\}$

The Serial mrOX GA

- The mrOX GA starts by first isolating a number of sub-populations for a several generations.
- Breeds new solutions using the mrOX crossover operator.
- Applies tour improvement heuristics like 2-opt and 1-swap on improved child solutions.
- Preserves diversity with a 5% chance of mutation.
- Terminates after the algorithm does not produce a better result in 150 generations.

Why Parallelize?

- Speedup
  - Provide higher quality solutions in less time.
- Increased Problem Size
  - Utilize more resources to attack larger problem instances.
- Robustness
  - Many serial heuristics require multiple input parameters that need to be tuned experimentally.
  - Each process can use a different set of parameters to avoid manual tuning.
  - Perform consistently on a range of problem instances.
- Cooperation
  - Use cooperative mechanisms to guide the search to more promising regions of the search space.

Cooperation Schemes

- No Cooperation
  - Provides a useful benchmark for testing other cooperation schemes.
- Solution Warehouse*
  - Workers periodically send solution updates to a central repository.
  - The repository synchronizes the workers to a set of the best solutions found so far.
- Inter-Worker Communication
  - Cooperation is structured on a specific topology.
  - Worker processes may only cooperate with their neighbors.
  - Example: Ring Topology

Classification of Parallel Meta-heuristics

- Three classifications from Crainic and Toulouse (2003)
- Type 1: Low-Level Parallelism
  - Attempts to speed up processing within an iteration of a heuristic method.
- Type 2: Partitioning of Solution Space
  - Partitions the solution space into subsets to explore in parallel.
- Type 3: Concurrent Exploration*
  - Multiple concurrent explorations of the solution space.

Parallel Approach to the GTSP

- Run multiple instances of the mrOX GA in parallel.
- The proposed architecture supports a type 3 classification: multiple concurrent explorations of the solution space.
- Implement the solution warehouse method of cooperation to guide worker processes to more promising regions of the search space.

Method of Approach

1. Develop a general parallel architecture for hosting sequential heuristic algorithms.*
2. Extend the framework provided by the architecture to host the mrOX GA and the GTSP problem class.
3. Implement the solution warehouse method of cooperation.

Implementation

- Initial Development and Validation:
  - Multi-processor PC running Linux O/S.
- Final Validation and Testing:
  - UMD's Deepthought Cluster, Linux O/S, up to 64 nodes with at least 2 processors.
- Language and Libraries:
  - C/C++
  - Message Passing Interface (MPI) Libraries
  - POSIX Threads Library

Database

- Based on a subset of TSP instances from the well-known TSPLib – a library of TSP instances.
- Use existing code for partitioning the nodes into clusters using method in (M. Fischetti, 1997).
- Use a set of larger instances tested in (Silberholz and Golden, 2007).
  - Number of nodes between 400 and 1084.
  - Number of clusters between 80 and 200.
  - The serial mrOX is already fast on small problem instances.
  - Don't have optimal results for larger instances but there are published results for tests of the mrOX GA and S&D GA on these instances.

Validation

1. Validate the parallel architecture by implementing a simple test algorithm with several test-cases.
2. Validate the parallel implementation of the mrOX GA using a single worker process.
3. Validate the parallel implementation of the mrOX GA using more than one worker process.

Testing

- Test how the parallel implementation scales with the number of processors.
- Use results (i.e. solution costs) from runs of the serial mrOX GA as a stopping criterion for the parallel implementation.
- Measure the run times while using different numbers of processors.
- Test the efficacy of the cooperation scheme using the no-cooperation scheme as a benchmark.
- Time permitting, try a different cooperation scheme.

References >

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